

# Package ‘PACLasso’

October 12, 2022

**Type** Package

**Title** Penalized and Constrained Lasso Optimization

**Version** 1.0.0

**Date** 2019-4-11

**Maintainer** Courtney Paulson <cpaulson@rhsmith.umd.edu>

**Description** An implementation of both the equality and inequality constrained lasso functions for the algorithm described in “Penalized and Constrained Optimization” by James, Paulson, and Rusmevichientong (Journal of the American Statistical Association, 2019; see <<http://www-bcf.usc.edu/~gareth/research/PAC.pdf>> for a full-text version of the paper).

The algorithm here is designed to allow users to define linear constraints (either equality or inequality constraints) and use a penalized regression approach to solve the constrained problem. The functions here are used specifically for constraints with the lasso formulation, but the method described in the PaC paper can be used for a variety of scenarios. In addition to the simple examples included here with the corresponding functions, complete code to entirely reproduce the results of the paper is available online through the Journal of the American Statistical Association.

**Depends** R (>= 3.3.0), methods (>= 3.4.4), penalized (>= 0.9)

**Imports** MASS (>= 7.3), lars (>= 1.2), quadprog (>= 1.5), limSolve (>= 1.5.5.3)

**License** GPL-3

**URL** <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>

**Repository** CRAN

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 6.1.1

**NeedsCompilation** no

**Author** Courtney Paulson [aut, cre],  
Gareth James [ctb],  
Paat Rusmevichientong [ctb]

**Date/Publication** 2019-04-29 21:30:16 UTC

## R topics documented:

generate.data . . . . .	2
lars.c . . . . .	3
lars.ineq . . . . .	5
lasso.c . . . . .	6
lasso.ineq . . . . .	8
lin.int . . . . .	9
lin.int.ineq . . . . .	10
quad.int . . . . .	10
quad.int.ineq . . . . .	11
transformed . . . . .	12
transformed.ineq . . . . .	13
<b>Index</b>	<b>15</b>

---

generate.data	<i>Function to Randomly Generate Data (with Constraints)</i>
---------------	--

---

### Description

This function is primarily used for reproducibility. It will generate a data set of a given size with a given number of constraints for testing function code.

### Usage

```
generate.data(n = 1000, p = 10, m = 5, cov.mat = NULL, s = 5,
             sigma = 1, glasso = F, err = 0)
```

### Arguments

n	number of rows in randomly-generated data set (default is 1000)
p	number of variables in randomly-generated data set (default is 10)
m	number of constraints in randomly-generated constraint matrix (default is 5)
cov.mat	a covariance matrix applied in the generation of data to impose a correlation structure. Default is NULL (no correlation)
s	number of true non-zero elements in coefficient vector beta1 (default is 5)
sigma	standard deviation of noise in response (default is 1, indicating standard normal)
glasso	should the generalized Lasso be used (TRUE) or standard Lasso (FALSE). Default is FALSE
err	error to be introduced in random generation of coefficient values. Default is no error (err = 0)

**Value**

x generated x data  
 y generated response y vector  
 C.full generated full constraint matrix (with constraints of the form  $C.full * beta = b$ )  
 b generated constraint vector b  
 b.run if error was included, the error-adjusted value of b  
 beta the complete beta vector, including generated beta1 and beta2

**References**

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
dim(random_data$x)
head(random_data$y)
dim(random_data$C.full)
random_data$beta
```

---

 lars.c

---

*Constrained LARS Coefficient Function (Equality Constraints)*


---

**Description**

This function computes the PaC constrained LASSO coefficient paths following the methodology laid out in the PaC paper. This function could be called directly as a standalone function, but the authors recommend using `lasso.c` for any implementation. This is because `lasso.c` has additional checks for errors across the coefficient paths and allows for users to go forwards and backwards through the paths if the paths are unable to compute in a particular direction for a particular run.

**Usage**

```
lars.c(x, y, C.full, b, l.min = -2, l.max = 6, step = 0.2,
      beta0 = NULL, verbose = F, max.it = 12, intercept = T,
      normalize = T, forwards = T)
```

**Arguments**

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with constraints of the form $C.full * beta = b$ )
b	constraint vector b

l.min	lowest value of lambda to consider (used as $10^{l.min}$ ). Default is -2
l.max	largest value of lambda to consider (used as $10^{l.max}$ ). Default is 6
step	step size increase in lambda attempted at each iteration (by a factor of $10^{step}$ ). Default is 0.2
beta0	initial guess for beta coefficient vector. Default is NULL (indicating initial vector should be calculated by algorithm)
verbose	should function print output at each iteration (TRUE) or not (FALSE). Default is FALSE
max.it	maximum number of times step size is halved before the algorithm terminates and gives a warning. Default is 12
intercept	should intercept be included in modeling (TRUE) or not (FALSE). Default is TRUE.
normalize	should x data be normalized. Default is TRUE
forwards	if forwards = F, then the algorithm starts at $10^{l.max}$ and moves backwards (without the forward step). If forwards = T, algorithm starts at $10^{l.min}$ and works forward. Default is FALSE

### Value

coefs A p by length(lambda) matrix with each column corresponding to the beta estimate for that lambda

lambda the grid of lambdas used to calculate the coefficients on the coefficient path

intercept vector with each element corresponding to intercept for corresponding lambda

error did the algorithm terminate due to too many iterations (TRUE or FALSE)

b2index the index of the beta2 values identified by the algorithm at each lambda

### References

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

### Examples

```

random_data = generate.data(n = 500, p = 20, m = 10)
lars_fit = lars.c(random_data$x, random_data$y, random_data$C.full, random_data$b)
lars_fit$lambda
lars_fit$error
### The coefficients for the first lambda value
lars_fit$coefs[1,]
### Example of code where path is unable
### to be finished (only one iteration)
lars_err = lars.c(random_data$x, random_data$y, random_data$C.full,
random_data$b, max.it = 1)
lars_err$error
lars_err$lambda

```

**Description**

This function computes the PaC constrained LASSO coefficient paths following the methodology laid out in the PaC paper but with inequality constraints. This function could be called directly as a standalone function, but the authors recommend using `lasso.ineq` for any implementation. This is because `lasso.ineq` has additional checks for errors across the coefficient paths and allows for users to go forwards and backwards through the paths if the paths are unable to compute in a particular direction for a particular run.

**Usage**

```
lars.ineq(x, y, C.full, b, l.min = -2, l.max = 6, step = 0.2,
         beta0 = NULL, verbose = F, max.it = 12, intercept = T,
         normalize = T, forwards = T)
```

**Arguments**

<code>x</code>	independent variable matrix of data to be used in calculating PaC coefficient paths
<code>y</code>	response vector of data to be used in calculating PaC coefficient paths
<code>C.full</code>	complete inequality constraint matrix $C$ (with inequality constraints of the form $C.full * beta \geq b$ )
<code>b</code>	constraint vector $b$
<code>l.min</code>	lowest value of lambda to consider (used as $10^{l.min}$ ). Default is -2
<code>l.max</code>	largest value of lambda to consider (used as $10^{l.max}$ ). Default is 6
<code>step</code>	step size increase in lambda attempted at each iteration (by a factor of $10^{step}$ ). Default is 0.2
<code>beta0</code>	initial guess for beta coefficient vector. Default is NULL (indicating initial vector should be calculated by algorithm)
<code>verbose</code>	should function print output at each iteration (TRUE) or not (FALSE). Default is FALSE
<code>max.it</code>	maximum number of times step size is halved before the algorithm terminates and gives a warning. Default is 12
<code>intercept</code>	should intercept be included in modeling (TRUE) or not (FALSE). Default is TRUE.
<code>normalize</code>	should x data be normalized. Default is TRUE
<code>forwards</code>	if <code>forwards = F</code> , then the algorithm starts at $10^{l.max}$ and moves backwards (without the forward step). If <code>forwards = T</code> , algorithm starts at $10^{l.min}$ and works forward. Default is FALSE

**Value**

coefs A p by length(lambda) matrix with each column corresponding to the beta estimate for that lambda

lambda the grid of lambdas used to calculate the coefficients on the coefficient path

intercept vector with each element corresponding to intercept for corresponding lambda

error did the algorithm terminate due to too many iterations (TRUE or FALSE)

b2index the index of the beta2 values identified by the algorithm at each lambda

**References**

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
lars_fit = lars.ineq(random_data$x, random_data$y, random_data$C.full, random_data$b)
lars_fit$lambda
lars_fit$error
### The coefficients for the first lambda value
lars_fit$coefs[1,]
### Example of code where path is unable to be finished
### (only one iteration)
lars_err = lars.ineq(random_data$x, random_data$y, random_data$C.full,
random_data$b, max.it = 1)
lars_err$error
lars_err$lambda
```

---

lasso.c

*Complete Run of Constrained LASSO Path Function (Equality Constraints)*

---

**Description**

This is a wrapper function for the `lars.c` PaC constrained Lasso function. `lasso.c` controls the overall path, providing checks for the path and allowing the user to control how the path is computed (and what to do in the case of a stopped path).

**Usage**

```
lasso.c(x, y, C.full, b, l.min = -2, l.max = 6, step = 0.2,
beta0 = NULL, verbose = F, max.it = 12, intercept = T,
normalize = T, backwards = F)
```

**Arguments**

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with constraints of the form $C.full * \beta = b$ )
b	constraint vector b
l.min	lowest value of lambda to consider (used as $10^{l.min}$ ). Default is -2
l.max	largest value of lambda to consider (used as $10^{l.max}$ ). Default is 6
step	step size increase in lambda attempted at each iteration (by a factor of $10^{step}$ ). Default is 0.2
beta0	initial guess for beta coefficient vector. Default is NULL (indicating initial vector should be calculated by algorithm)
verbose	should function print output at each iteration (TRUE) or not (FALSE). Default is FALSE
max.it	maximum number of times step size is halved before the algorithm terminates and gives a warning. Default is 12
intercept	should intercept be included in modeling (TRUE) or not (FALSE). Default is TRUE.
normalize	should X data be normalized. Default is TRUE
backwards	which direction should algorithm go, backwards from $\lambda = 10^{l.max}$ (TRUE) or forwards from $10^{l.max}$ and then backwards if algorithm gets stuck (FALSE). Default is FALSE.

**Value**

coefs A p by length(lambda) matrix with each column corresponding to the beta estimate for that lambda

lambda vector of values of lambda that were fit

intercept vector with each element corresponding to intercept for corresponding lambda

error Indicator of whether the algorithm terminated early because max.it was reached

**References**

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

**Examples**

```

random_data = generate.data(n = 500, p = 20, m = 10)
lasso_fit = lasso.c(random_data$x, random_data$y, random_data$C.full, random_data$b)
lasso_fit$lambda
lasso_fit$error
### The coefficients for the first lambda value
lasso_fit$coefs[1,]

```

```
### Example of code where path is unable to be finished
### (only one iteration), so both directions will be tried
lasso_err = lasso.c(random_data$x, random_data$y, random_data$C.full,
random_data$b, max.it = 1)
lasso_err$error
lasso_err$lambda
```

---

lasso.ineq	<i>Complete Run of Constrained LASSO Path Function with Inequality Constraints</i>
------------	--

---

### Description

This is a wrapper function for the `lars.c` PaC constrained Lasso function. `lasso.c` controls the overall path, providing checks for the path and allowing the user to control how the path is computed (and what to do in the case of a stopped path).

### Usage

```
lasso.ineq(x, y, C.full, b, l.min = -2, l.max = 6, step = 0.2,
beta0 = NULL, verbose = F, max.it = 12, intercept = T,
normalize = T, backwards = F)
```

### Arguments

<code>x</code>	independent variable matrix of data to be used in calculating PaC coefficient paths
<code>y</code>	response vector of data to be used in calculating PaC coefficient paths
<code>C.full</code>	complete constraint matrix C (with inequality constraints of the form $C.full * \beta \geq b$ )
<code>b</code>	constraint vector b
<code>l.min</code>	lowest value of lambda to consider (used as $10^{l.min}$ ). Default is -2
<code>l.max</code>	largest value of lambda to consider (used as $10^{l.max}$ ). Default is 6
<code>step</code>	step size increase in lambda attempted at each iteration (by a factor of $10^{step}$ ). Default is 0.2
<code>beta0</code>	initial guess for beta coefficient vector. Default is NULL (indicating initial vector should be calculated by algorithm)
<code>verbose</code>	should function print output at each iteration (TRUE) or not (FALSE). Default is FALSE
<code>max.it</code>	maximum number of times step size is halved before the algorithm terminates and gives a warning. Default is 12
<code>intercept</code>	should intercept be included in modeling (TRUE) or not (FALSE). Default is TRUE.
<code>normalize</code>	should X data be normalized. Default is TRUE
<code>backwards</code>	which direction should algorithm go, backwards from $10^{l.max}$ (TRUE) or forwards from $10^{l.max}$ and then backwards if algorithm gets stuck (FALSE). Default is FALSE.



**Value**

coefs A p by length(lambda) matrix with each column corresponding to the beta estimate for that lambda

lambda vector of values of lambda that were fit

intercept vector with each element corresponding to intercept for corresponding lambda

error Indicator of whether the algorithm terminated early because max.it was reached

**References**

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
lasso_fit = lasso.ineq(random_data$x, random_data$y, random_data$C.full, random_data$b)
lasso_fit$lambda
lasso_fit$error
### The coefficients for the first lambda value
lasso_fit$coefs[1,]
### Example of code where path is unable to be finished
### (only one iteration), so both directions will be tried
lasso_err = lasso.ineq(random_data$x, random_data$y, random_data$C.full,
random_data$b, max.it = 1)
lasso_err$error
lasso_err$lambda
```

---

lin.int

---

*Initialize Linear Programming Fit (Equality Constraints)*


---

**Description**

This function is called internally by `lars.c` to get the linear programming initial fit if the user requests implementation of the algorithm starting at the largest lambda value and proceeding backwards.

**Usage**

```
lin.int(C.full, b)
```

**Arguments**

C.full complete constraint matrix C (with constraints of the form  $C.full * beta = b$ )  
b constraint vector b

**Value**

beta the initial beta vector of coefficients to use for the PaC algorithm

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
lin_start = lin.int(random_data$C.full, random_data$b)
lin_start
```

---

lin.int.ineq

*Initialize Linear Programming Fit with Inequality Constraints*


---

**Description**

This function is called internally by `lars.ineq` to get the linear programming initial fit if the user requests implementation of the algorithm starting at the largest lambda value and proceeding backwards.

**Usage**

```
lin.int.ineq(C.full, b)
```

**Arguments**

<code>C.full</code>	complete constraint matrix C (with inequality constraints of the form $C.full * \beta \geq b$ )
<code>b</code>	constraint vector b

**Value**

beta the initial beta vector of coefficients to use for the PaC algorithm

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
lin_start = lin.int.ineq(random_data$C.full, random_data$b)
lin_start
```

---

quad.int

*Initialize Quadratic Programming Fit (Equality Constraints)*


---

**Description**

This function is called internally by `lars.c` to get the quadratic programming fit if the user requests implementation of the algorithm starting at the smallest lambda value and proceeding forwards.

**Usage**

```
quad.int(x, y, C.full, b, lambda, d = 10^-7)
```

**Arguments**

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with constraints of the form $C.full * \beta = b$ )
b	constraint vector b
lambda	value of lambda
d	very small diagonal term to allow for SVD (default $10^{-7}$ )

**Value**

beta the initial beta vector of coefficients to use for the PaC algorithm

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
quad_start = quad.int(random_data$x, random_data$y, random_data$C.full,
random_data$b, lambda = 0.01)
quad_start
```

---

quad.int.ineq

---

*Initialize Quadratic Programming Fit with Inequality Constraints*


---

**Description**

This function is called internally by `lars.ineq` to get the quadratic programming fit if the user requests implementation of the algorithm starting at the smallest lambda value and proceeding forwards.

**Usage**

```
quad.int.ineq(x, y, C.full, b, lambda, d = 10-5)
```

**Arguments**

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with inequality constraints of the form $C.full * \beta \geq b$ )
b	constraint vector b
lambda	value of lambda
d	very small diagonal term to allow for SVD (default $10^{-7}$ )

**Value**

beta the initial beta vector of coefficients to use for the PaC algorithm

**Examples**

```
random_data = generate.data(n = 500, p = 20, m = 10)
quad_start = quad.int.ineq(random_data$x, random_data$y,
  random_data$C.full, random_data$b, lambda = 0.01)
quad_start
```

---

transformed

---

*Transform Data to Fit PaC Implementation (Equality Constraints)*


---

**Description**

This function is called internally by `lars.c` to compute the transformed versions of the X, Y, and constraint matrix data, as shown in the PaC paper.

**Usage**

```
transformed(x, y, C.full, b, lambda, beta0, eps = 10^-8)
```

**Arguments**

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with constraints of the form $C.full * beta = b$ )
b	constraint vector b
lambda	value of lambda
beta0	initial guess for beta coefficient vector
eps	value close to zero used to verify SVD decomposition. Default is $10^{-8}$

**Value**

x transformed x data to be used in the PaC algorithm  
y transformed y data to be used in the PaC algorithm  
Y\_star transformed  $Y^*$  value to be used in the PaC algorithm  
a2 index of A used in the calculation of beta2 (the non-zero coefficients)  
beta1 beta1 values  
beta2 beta2 values  
C constraint matrix  
C2 subset of constraint matrix corresponding to non-zero coefficients  
active.beta index of non-zero coefficient values  
beta2.index index of non-zero coefficient values

## References

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

## Examples

```
random_data = generate.data(n = 500, p = 20, m = 10)
transform_fit = transformed(random_data$x, random_data$y, random_data$C.full,
random_data$b, lambda = 0.01, beta0 = rep(0,20))
dim(transform_fit$x)
head(transform_fit$y)
dim(transform_fit$C)
transform_fit$active.beta
```

---

transformed.ineq      *Transform Data to Fit PaC Implementation for Inequality Constraints*

---

## Description

This function is called internally by `lars.c` to compute the transformed versions of the X, Y, and constraint matrix data, as shown in the PaC paper.

## Usage

```
transformed.ineq(x, y, C.full, b, lambda, beta0, eps = 10^-8)
```

## Arguments

x	independent variable matrix of data to be used in calculating PaC coefficient paths
y	response vector of data to be used in calculating PaC coefficient paths
C.full	complete constraint matrix C (with inequality constraints of the form $C.full * beta \geq b$ )
b	constraint vector b
lambda	value of lambda
beta0	initial guess for beta coefficient vector
eps	value close to zero used to verify SVD decomposition. Default is $10^{-8}$

## Value

x transformed x data to be used in the PaC algorithm  
y transformed y data to be used in the PaC algorithm  
Y\_star transformed Y\* value to be used in the PaC algorithm  
a2 index of A used in the calculation of beta2 (the non-zero coefficients)

beta1 beta1 values  
beta2 beta2 values  
C constraint matrix  
C2 subset of constraint matrix corresponding to non-zero coefficients  
active.beta index of non-zero coefficient values  
beta2.index index of non-zero coefficient values

## References

Gareth M. James, Courtney Paulson, and Paat Rusmevichientong (JASA, 2019) "Penalized and Constrained Optimization." (Full text available at <http://www-bcf.usc.edu/~gareth/research/PAC.pdf>)

## Examples

```
random_data = generate.data(n = 500, p = 20, m = 10)
transform_fit = transformed.ineq(random_data$x, random_data$y,
random_data$C.full, random_data$b, lambda = 0.01, beta0 = rep(0,20))
dim(transform_fit$x)
head(transform_fit$y)
dim(transform_fit$C)
transform_fit$active.beta
```

# Index

`generate.data`, [2](#)

`lars.c`, [3](#)

`lars.ineq`, [5](#)

`lasso.c`, [6](#)

`lasso.ineq`, [8](#)

`lin.int`, [9](#)

`lin.int.ineq`, [10](#)

`quad.int`, [10](#)

`quad.int.ineq`, [11](#)

`transformed`, [12](#)

`transformed.ineq`, [13](#)