

# Package ‘RationalMatrix’

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**Type** Package

**Title** Exact Matrix Algebra for Rational Matrices

**Version** 1.0.0

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**Description** Provides functions to deal with matrix algebra for matrices with rational entries: determinant, rank, image and kernel, inverse, Cholesky decomposition. All computations are exact.

**License** GPL-3

**URL** <https://github.com/stla/RationalMatrix>

**BugReports** <https://github.com/stla/RationalMatrix/issues>

**Imports** gmp, Rcpp (>= 1.0.9)

**LinkingTo** BH, Rcpp, RcppEigen

**Encoding** UTF-8

**RoxygenNote** 7.2.3

**SystemRequirements** C++ 17, gmp

**NeedsCompilation** yes

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**Repository** CRAN

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QcholUtDU	<i>'UtDU' decomposition of a rational matrix</i>
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### Description

Cholesky-'UtDU' decomposition of a symmetric rational matrix.

### Usage

```
QcholUtDU(M)
```

### Arguments

M a square matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{i,j}$

### Value

The Cholesky-'UtDU' decomposition of M in a list (see example).

### Note

Symmetry is not checked! Only the lower triangular part of M is used.

### Examples

```
library(RationalMatrix)
x <- matrix(c(1:5, (1:5)^2), 5, 2)
x <- cbind(x, x[, 1L] + 3L*x[, 2L])
M <- crossprod(x)
UtDU <- QcholUtDU(M)
library(gmp)
U <- as.bigq(UtDU$U)
D <- matrix("0", 3L, 3L)
diag(D) <- UtDU$D
D <- as.bigq(D)
perm <- UtDU$perm
UP <- U[, perm]
t(UP) %*% D %*% UP # this is `M`
```

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Qdet	<i>Determinant of a rational matrix</i>
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**Description**

Determinant of a square matrix with rational entries.

**Usage**

Qdet(M)

**Arguments**

M a square matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{i,j}$

**Value**

A string: quoted rational number representing the determinant.

**Examples**

```
library(RationalMatrix)
M <- cbind(c("1/2", "3"), c("5/3", "-2/7"))
Qdet(M)
```

---

Qinverse	<i>Inverse of a rational matrix</i>
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**Description**

Inverse matrix of a square rational matrix.

**Usage**

Qinverse(M)

**Arguments**

M a square matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{i,j}$

**Value**

A character matrix representing the inverse of M.

**Examples**

```
library(RationalMatrix)
M <- cbind(c("1/2", "3", "1"), c("5/3", "-2/7", "10/3"), c("0", "1", "2"))
Qinverse(M)
```

---

QisInjective                      *Check injectivity*

---

**Description**

Checks whether a rational matrix represents an injective linear map (i.e. has trivial kernel).

**Usage**

```
QisInjective(M)
```

**Arguments**

M                      a matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{i,j}$

**Value**

A Boolean value indicating whether the linear map corresponding to M is injective.

**Examples**

```
library(RationalMatrix)
set.seed(666L)
M <- matrix(rpois(35L, 1), 5L, 7L)
QisInjective(M)
```

---

QisInvertible                      *Check invertibility*

---

**Description**

Checks whether a square rational matrix is invertible.

**Usage**

```
QisInvertible(M)
```

**Arguments**

M                      a square matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{i,j}$

**Value**

A Boolean value indicating whether M is invertible.

**Examples**

```
library(RationalMatrix)
set.seed(666L)
M <- matrix(rpois(25L, 1), 5L, 5L)
QisInvertible(M)
```

---

QisSurjective	<i>Check surjectivity</i>
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**Description**

Checks whether a rational matrix represents a surjective linear map.

**Usage**

```
QisSurjective(M)
```

**Arguments**

M a matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{ij}$

**Value**

A Boolean value indicating whether the linear map corresponding to M is surjective.

**Examples**

```
library(RationalMatrix)
set.seed(666L)
M <- matrix(rpois(35L, 1), 7L, 5L)
QisSurjective(M)
```

---

Qkernel *Kernel of a rational matrix*

---

**Description**

Kernel (null-space) of a rational matrix.

**Usage**

Qkernel(M)

**Arguments**

M a matrix such that as.character(M<sub>ij</sub>) is a quoted integer or a quoted fraction for each entry M<sub>ij</sub>

**Value**

A character matrix representing a basis of the kernel of M. Note that this basis is not orthogonal.

**Examples**

```
library(RationalMatrix)
set.seed(666L)
M <- matrix(rpois(30L, 6), 10L, 3L)
M <- cbind(M, M[,1] + M[,2], M[,2] + 2L*M[,3])
Qkernel(M)
```

---

Qrange *Range of a rational matrix*

---

**Description**

Range (column-space, image, span) of a rational matrix.

**Usage**

Qrange(M)

**Arguments**

M a matrix such that as.character(M<sub>ij</sub>) is a quoted integer or a quoted fraction for each entry M<sub>ij</sub>

**Value**

A character matrix representing a basis of the range of M. Note that this basis is not orthogonal.

**Examples**

```
library(RationalMatrix)
set.seed(666L)
M <- matrix(rpois(15L, 6), 3L, 5L)
Qrange(M)
```

---

Qrank

*Rank of a rational matrix*

---

**Description**

Returns the rank of a rational matrix.

**Usage**

```
Qrank(M)
```

**Arguments**

M a matrix such that `as.character(M[i,j])` is a quoted integer or a quoted fraction for each entry  $M_{ij}$

**Value**

An integer, the rank of M.

**Examples**

```
library(RationalMatrix)
M <- cbind(c("1/2", "3", "1"), c("5/3", "-2/7", "10/3"), c("1", "1", "2"))
Qrank(M)
```

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